G9-5 investigation Name ___________________________________

Conditional Probability

**Example 1:**
Consider a high school with 400 students. The table shows the number of students in each class who earned different letter grades. A student is selected randomly.

| | Freshman | Sophomores | Juniors | Seniors | TOTAL |
|---|---|---|---|---|---|
| **A** | 20 | 18 | 25 | 18 | **81** |
| **B** | 30 | 40 | 33 | 50 | **153** |
| **C** | 30 | 17 | 20 | 20 | **87** |
| **D** | 15 | 15 | 12 | 4 | **46** |
| **F** | 5 | 20 | 5 | 3 | **33** |
| **TOTAL** | **100** | **110** | **95** | **95** | **400** |

**a)** P(freshman) = **b)** P(earned an A)=

Now suppose you select a **random freshman** from the school. What is the probability that they got an A?

When you have additional information that causes you to restrict the sample space you are considering, you are working with **conditional probability**.

We write this as P(earned an A|freshman) =

We say, "the probability the student earned an A, **given** it is the freshman".

Explain why P(earned an A) ≠ P(earned an A|freshman).

Find each conditional probability.

**a)** P(earned a C | Junior)= **b)** P(Junior | earned a C)=

**c)** P(earned a C or higher | Senior)= **d)** P(Freshman | earned a D or F)=

Sometimes we do not have a handy chart like the one above. We can still calculate conditional probability using:

$$\boxed{\mathbf{P(A|B)} = \frac{\boldsymbol{P(A \cap B)}}{\mathbf{P(B)}}}$$

Let's test it: $\dfrac{\text{P(earned an A} \cap \text{freshman)}}{\text{P(freshman)}} =$

**Example 2:** 44% of the teachers at Frankline High School are male, 25% of the teachers are math teachers, and 12% are male math teachers.

a) P(male) =

b) P(male $\cap$ math teacher)=

c) P(male | math teacher) =

Note, P(male) $\neq$ P(male | math teacher). When this occurs, we say that the events {male} and {math teacher} are **associated**.

**Example 3:** Data was collected regarding how many students were wearing black shirts, hats, or both.

**a)** Find P(hat) =

**b)** Find P(hat | black shirt) =

| | **Black Shirt** | **No Black Shirt** | **Total** |
|---|---|---|---|
| **Hat** | 4 | 3 | 7 |
| **No Hat** | 16 | 12 | 28 |
| **Total** | 20 | 15 | 35 |

Note that these probabilities are the same. The probability that a student is wearing a hat is the same, regardless of whether or not we know that they are wearing a black shirt. In this situation, we say that the events {hat} and {black shirt} are **independent.**

**INDEPENDENT vs. ASSOCIATED EVENTS**

**If P(A) $\neq$ P(A | B), the events A and B are <u>associated</u>.**

**If P(A) = P(A | B), the events A and B are <u>independent</u>.**

Two standard dice are rolled.

a.) P(sum of 12) =

b.) P(sum of 12 $\cap$ doubles) =

c.) P(sum of 12 | doubles)

d.) Are the events {sum of 12} and {doubles} associated or independent? Explain.